VCA EduTech and AI Private Limited

AI-enabled coaching material for NEET & IIT-JEE

JEE MAIN & ADVANCED · STUDY MODULE

Physics · Class 12

Electric Charges and Fields

Theory · Illustrations · Try Yourself · Summary · Section A (Main) · Section B (Advanced) · Section C

7	26	35+	3
Topics	Colour figures	Try-yourself	Graded sections



MEET YOUR MENTOR

Vijay K. Sharma (VS Sir)

Founder & Director · Author

Ex-Indian Navy Naval Artificer · 20+ years of teaching NEET & JEE Physics · AI-enabled classroom methods

VCA Books & Chapter Bundles

Chapter-wise NEET & JEE modules for Physics, Chemistry, Biology and Maths · Test Series (CBT) · School Integrated Program · Home Tuition

vcakota.com · +91 96024 05311

Edition 2026-27 · Jaipur, Rajasthan, India · vcaedutechai@gmail.com

Original study material of VCA EduTech and AI Private Limited, Jaipur. For enrolled students only. No part may be copied, shared or resold without written permission.

Contents

1. Electric charge and its properties
2. Coulomb's law and superposition
3. Electric field and field lines
4. Electric dipole
5. Electric flux and Gauss's law
6. Applications of Gauss's law
7. Continuous charge distributions (ring, disc, arc, rod)
8. Chapter summary
9. Section A: JEE Main level questions
10. Section B: JEE Advanced pattern and previous-year-pattern questions
11. Section C: Challenge problems (HCV / Irodov style, original)
12. Answer keys and solutions

How to study this module. Read each topic, mark the TRICK and EXAM ALERT boxes, and derive every JEE-extension result once by yourself. Do the Illustrations before reading solutions and finish each TRY YOURSELF box before moving on. Section A has JEE Main-level questions, Section B has JEE Advanced-pattern questions (multiple-correct, match, passage and integer type) and Section C collects questions framed on the pattern of previous Main and Advanced papers. Use $g = 10\ \mathrm{m\,s^{-2}}$, $k = 9 \times 10^9\ \mathrm{N\,m^2C^{-2}}$ unless told otherwise.

1 Electric charge and its properties

Charge is the property of matter due to which it experiences a force in an electric or magnetic field. It is a **scalar** with SI unit **coulomb (C)**. Matter is electrically neutral because it contains equal numbers of protons and electrons. A body becomes **positively charged** by *losing* electrons and **negatively charged** by *gaining* electrons. Only electrons move; protons stay locked in the nucleus.

Particle	Charge	Mass
Electron	-1.6×10^{-19} C	9.1×10^{-31} kg
Proton	$+1.6 \times 10^{-19}$ C	1.67×10^{-27} kg
Neutron	0	1.675×10^{-27} kg

Properties of charge

1. **Additivity:** total charge = algebraic sum of all charges.
2. **Conservation:** charge can be neither created nor destroyed; it only moves from one body to another (pair production creates $+e$ and $-e$ together, net zero).
3. **Quantisation:** $q = \pm ne$ with $n = 1, 2, 3\ldots$ The charge $e = 1.6 \times 10^{-19}$ C is the least free charge. (Quarks carry $\pm e/3$, $\pm 2e/3$ but are never found free.)
4. **Invariance:** charge does not change with speed, unlike mass.

Methods of charging

- **Friction:** two different materials rubbed together transfer electrons; both acquire equal and opposite charges (glass rubbed with silk: glass $+$, silk $-$; ebonite rubbed with fur: ebonite $-$).
- **Conduction (contact):** a charged body touches a conductor; the conductor gets the *same kind* of charge.
- **Induction:** a charged body is only *brought near* a conductor; the near end gets the *opposite* charge, the far end the *same*. If the far end is earthed and then the earth connection is removed before the rod, the body is left with the opposite charge.

Conductors (metals, human body, earth) allow charge to flow; insulators (glass, plastic, rubber) do not. The **gold-leaf electroscope** detects charge: leaves diverge when charged, and the divergence increases when a like charge is brought near.

JEE extension

- Charge sharing between two conductors in contact: potentials equalise, so charges divide in proportion to radii, $\frac{q_1}{q_2} = \frac{R_1}{R_2}$ (not equally unless the spheres are identical).
- Repeated touching with identical neutral spheres halves the charge each time: after n contacts $q_n = q/2^n$.
- Charge is relativistically invariant; mass is not. Nuclear reactions conserve charge and nucleon number.

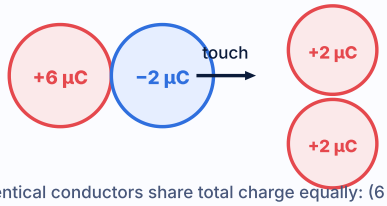
⚡ **TRICK** Number of electrons in charge q: $n = \frac{q}{e} = \frac{q}{1.6 \times 10^{-19}}$. One coulomb contains 6.25×10^{18} electrons. A body that gains n electrons also gains mass nm_e.

⚠ **EXAM ALERT** A charged body is **never** a fraction of e. If an option gives $1.5e$ or 2.4×10^{-19} C, reject it. A positively charged body has slightly *less* mass; a negatively charged body slightly *more*.



Induction gives the OPPOSITE sign, no contact

Fig. 1.1 Charging by induction



Identical conductors share total charge equally: (6 − 2)/2 = +2 µC each

Fig. 1.2 Charge sharing between identical conductors

Illustration 1.1

A body carries 2.5×10^{13} excess electrons. Find its charge and the mass gained.

Solution. $q = ne = 2.5 \times 10^{13} \times 1.6 \times 10^{-19} = 4 \times 10^{-6}$ C $= -4\,\mu$C. Mass gained $= nm_e = 2.5 \times 10^{13} \times 9.1 \times 10^{-31} = 2.3 \times 10^{-17}$ kg.

Illustration 1.2

How many electrons make up 1 C of charge, and what is their mass?

Solution. $n = \dfrac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$. Mass $= 6.25 \times 10^{18} \times 9.1 \times 10^{-31} = 5.7 \times 10^{-12}$ kg.

TRY YOURSELF 1

1. Two conducting spheres of radii 2 cm and 4 cm share $12\,\mu$C when connected by a wire. The charges on them are:
   (A) $6\,\mu$C and $6\,\mu$C **(B)** $3\,\mu$C and $9\,\mu$C **(C)** $4\,\mu$C and $8\,\mu$C **(D)** $8\,\mu$C and $4\,\mu$C

2. A body of mass 1 g loses 10^{13} electrons. The fractional change in its mass is:
   (A) zero **(B)** 9.1×10^{-15} (increase) **(C)** 9.1×10^{-18} (decrease) **(D)** 9.1×10^{-15} (decrease)

3. Which of these is NOT invariant with the speed of the body?
   (A) mass **(B)** sign of charge **(C)** charge **(D)** number of electrons

4. A charged sphere A (charge q) touches identical neutral spheres B, C, D one after another. The final charge on A is:
   (A) $q/3$ **(B)** $q/8$ **(C)** $q/2$ **(D)** $q/4$

2 Coulomb's law and superposition

Coulomb's law. The force between two stationary point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance, acting along the line joining them:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = k\frac{q_1 q_2}{r^2}, \qquad k = 9 \times 10^9\ \mathrm{N\,m^2 C^{-2}},\ \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2 N^{-1} m^{-2}}$$

In vector form $\vec{F}_{12} = k\dfrac{q_1 q_2}{r^2}\hat{r}_{12}$ is the force on q_1 due to q_2. Like charges repel, unlike attract, and $\vec{F}_{12} = -\vec{F}_{21}$.

In a medium. $F_{\text{medium}} = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q_1 q_2}{r^2} = \dfrac{F_{\text{air}}}{K}$, where $K = \varepsilon_r$ is the dielectric constant ($K = 1$ for air/vacuum, $K = 80$ for water, $K = \infty$ for a conductor).

Superposition principle. The force on a charge due to several charges is the *vector sum* of the individual forces. Pairwise forces are unaffected by other charges.

Electric force	Gravitational force
Attractive or repulsive	Always attractive
Depends on the medium	Independent of the medium
$F_e/F_g \approx 10^{36}$ for two protons	Weakest of the fundamental forces
Both are central, conservative and follow the inverse-square law.	

JEE extension

- Vector form with position vectors: $\vec{F}_{12} = \dfrac{kq_1 q_2(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$.
- **Earnshaw's idea:** a charge cannot be in *stable* equilibrium under electrostatic forces alone. Three collinear charges in equilibrium are unstable along the line (but may be stable perpendicular to it).
- **Hanging balls:** $\tan\theta = \dfrac{kq^2}{mg(2L\sin\theta)^2}$; for small θ the separation is $x = \left(\dfrac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}$ and $x \propto q^{2/3}$.

- Equilibrium of four charges, square or tetrahedral arrangements: balance the outward repulsion against the central charge's pull.

⚡ TRICK **Null point of two charges** (distance d apart, $q_1 > q_2$ in magnitude). For *like* charges the null point lies between them at $x = \dfrac{d\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}$ from q_1. For *unlike* charges it lies outside, beyond the smaller charge, at $x = \dfrac{d\sqrt{q_2}}{\sqrt{q_1}-\sqrt{q_2}}$ from q_2.

⚡ TRICK **Equilibrium of a third charge.** For two like charges q_1, q_2, a third charge placed at the null point is in equilibrium if $q_3 = -\dfrac{q_1 q_2}{(\sqrt{q_1}+\sqrt{q_2})^2}$. For $q_1 = q_2 = q$ this gives $q_3 = -q/4$ at the midpoint.

⚡ TRICK **Two identical hanging spheres in a liquid.** If the angle between the threads is the same in air and in a liquid of density σ and dielectric constant K, then the density of the sphere material is $\rho = \dfrac{K\sigma}{K-1}$ (since $K(1-\sigma/\rho) = 1$).

⚠ EXAM ALERT When identical conducting spheres are touched, the total charge is shared **equally**. The force may change in size and even in nature (attractive to repulsive).

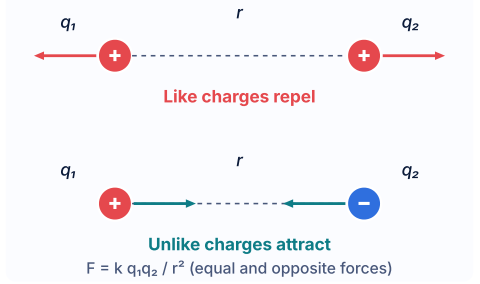


Fig. 2.1 Forces between two point charges

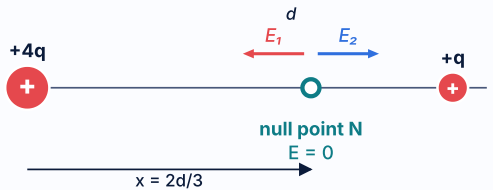


Fig. 2.2 Null point between +4q and +q

Illustration 2.1

Two charges $+5\,\mu\text{C}$ and $-3\,\mu\text{C}$ are 0.3 m apart in air. Find the force between them.

Solution. $F = \dfrac{9\times10^9 \times 5\times10^{-6}\times 3\times10^{-6}}{(0.3)^2} = 1.5$ N (attractive).

Illustration 2.2

Charges $+9\,\mu\text{C}$ and $+4\,\mu\text{C}$ are 50 cm apart. Locate the point on the line joining them where the field is zero.

Solution. $x = \dfrac{d\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}} = \dfrac{0.5\times 3}{3+2} = 0.3$ m from the $9\,\mu$C charge. Check: $\dfrac{9}{0.3^2} = \dfrac{4}{0.2^2} = 100$.

Illustration 2.3

Charges $+4q$ and $+q$ are d apart. Find the charge and position of a third charge that keeps all three in equilibrium.

Solution. Position (null point): $x = \dfrac{d\cdot 2\sqrt{q}}{2\sqrt{q}+\sqrt{q}} = \dfrac{2d}{3}$ from $4q$. Charge: $q_3 = -\dfrac{4q\cdot q}{(2\sqrt{q}+\sqrt{q})^2} = -\dfrac{4q}{9}$.

Illustration 2.4

Two identical spheres of density $1.5\ \text{g cm}^{-3}$ hang by threads. They keep the same angle between the threads when immersed in a liquid of density $0.5\ \text{g cm}^{-3}$. Find K of the liquid.

Solution. $K(1-\sigma/\rho) = 1 \Rightarrow K = \dfrac{\rho}{\rho-\sigma} = \dfrac{1.5}{1.0} = 1.5$.

TRY YOURSELF 2

1. A charge q is at the origin. $+Q$ is at $(a, 0)$ and $-Q$ at $(0, a)$. The force on q has magnitude:
   (A) $\frac{2kQq}{a^2}$ **(B)** $\frac{\sqrt{2}kQq}{a^2}$ **(C)** $\frac{kQq}{a^2}$ **(D)** zero

2. Two identical balls of mass 1 g and charge 20 nC hang from threads 1 m long. The separation for small angle is nearly:
   (A) 0.0448 m **(B)** 0.179 m **(C)** 0.0896 m **(D)** 0.0711 m

3. Charges $+q, +q, -q, -q$ are placed at the corners A, B, C, D (taken in order) of a square of side a. The force on the charge at A has magnitude:
   (A) $\frac{kq^2}{2a^2}$ **(B)** $\frac{\sqrt{2}kq^2}{a^2}$ **(C)** $\frac{kq^2}{a^2}$ **(D)** $\frac{3kq^2}{2a^2}$

4. Equal charges $+q$ at the vertices of an equilateral triangle of side a. A charge Q at the centroid keeps the system in equilibrium for $Q =$:
   (A) $-\frac{q}{\sqrt{3}}$ **(B)** $+\frac{q}{\sqrt{3}}$ **(C)** $-\frac{q}{3}$ **(D)** $-\frac{\sqrt{3}\,q}{2}$

3 Electric field and field lines

Electric field at a point is the force per unit positive test charge placed there (test charge so small it does not disturb the source):

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}, \qquad \text{unit: N C}^{-1}\ (= \text{V m}^{-1}),\ [E] = MLT^{-3}A^{-1}$$

For a point charge q: $E = \frac{kq}{r^2}$, directed away from $+q$ and towards $-q$. For many charges, $\vec{E} = \sum \vec{E}_i$ (vector sum).

Force and motion in a field. A charge q feels $\vec{F} = q\vec{E}$ (along $\vec{E}$ for $+q$, opposite for $-q$). A free charge released from rest in a uniform field moves in a straight line with $a = \frac{qE}{m}$. If it enters perpendicular to the field with speed v_0 it follows a parabola, just like a projectile:

$$y = \frac{1}{2}\frac{qE}{m}t^2, \quad t = \frac{x}{v_0} \Rightarrow y = \frac{qE\,x^2}{2mv_0^2}.$$

Field lines are imaginary curves whose tangent gives the direction of the field. (1) They start on $+$ charges and end on $-$ charges. (2) They never cross (two tangents would mean two directions). (3) They never form closed loops in electrostatics. (4) Their density (closeness) shows field strength. (5) They are perpendicular to the surface of a conductor. (6) They tend to contract along their length (attraction) and repel sideways (repulsion).

JEE extension

- **Charged pendulum.** In a horizontal field the bob settles at $\tan\theta = \frac{qE}{mg}$ and has $g_{\text{eff}} = \sqrt{g^2 + (qE/m)^2}$, so $T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$. In a vertical field $g_{\text{eff}} = g \pm \frac{qE}{m}$ (plus if the force on the bob is downward).
- **Deflection.** After length L of field: $\tan\phi = \frac{qEL}{mv_0^2}$, exit displacement $y = \frac{qEL^2}{2mv_0^2}$; extended backward the path appears to start from the midpoint of the plate.
- For continuous distributions use $dE = \frac{k\,dq}{r^2}$ and add components using symmetry (see Topic 7).

⚡ **TRICK** Field due to a system: if charges are arranged symmetrically (regular polygon of equal charges) the field at the centre is **zero**. Removing one charge q from such a polygon leaves a field $\frac{kq}{R^2}$ at the centre, directed *away* from the removed charge's position if q was positive.

⚠ **EXAM ALERT** Field lines show the path of a particle only if it starts from rest and the lines are straight. In a curved field the particle does not follow the line.

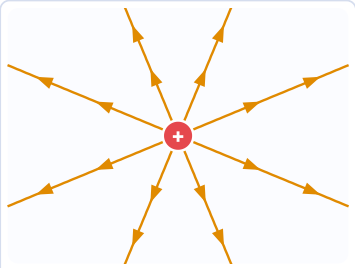


Fig. 3.1 Field of +q

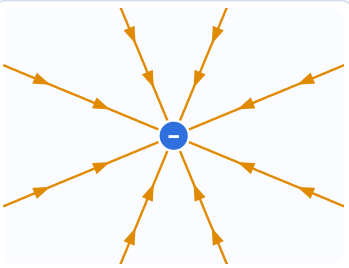


Fig. 3.2 Field of −q

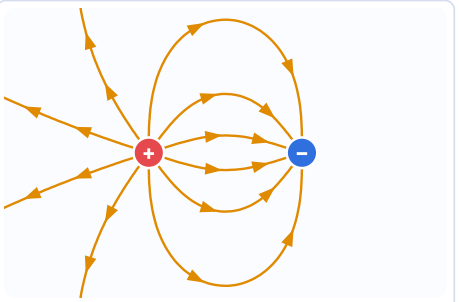


Fig. 3.3 Field lines of a dipole

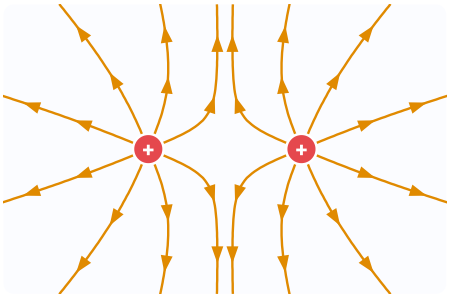


Fig. 3.4 Two equal positive charges

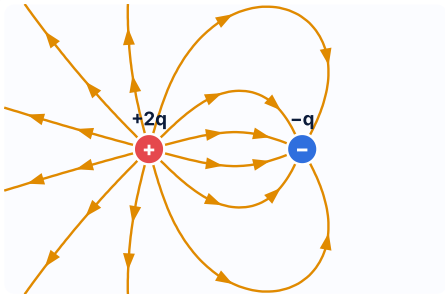


Fig. 3.5 Unequal charges: more lines from +2q

Illustration 3.1

Charges $+4\,\mu$C and $-4\,\mu$C are 20 cm apart. Find the field at the midpoint.

Solution. Both fields point from $+$ to $-$, so they add: $E = 2 \times \dfrac{9\times10^{9}\times4\times10^{-6}}{(0.1)^2} = 7.2\times10^{6}\ \mathrm{N\,C^{-1}}$.

Illustration 3.2

An electron enters a uniform field $E = 10^{3}\ \mathrm{N\,C^{-1}}$ at right angles with speed $2\times10^{6}\ \mathrm{m\,s^{-1}}$. The field region is 5 cm long. Find the deflection.

Solution. $a = \dfrac{eE}{m} = 1.76\times10^{14}\ \mathrm{m\,s^{-2}}$, $t = \dfrac{0.05}{2\times10^{6}} = 2.5\times10^{-8}$ s. $y = \tfrac{1}{2}at^2 = 0.0549\ \mathrm{m} \approx 5.5\ \mathrm{cm}$.

TRY YOURSELF 3

1. A bob of charge q is in a horizontal field with $qE = mg$. Compared with the period T_0 without a field, the period is:
   (A) $2^{-1/4}\,T_0 \approx 0.84\,T_0$ **(B)** $2^{1/4}\,T_0$ **(C)** $T_0/2$ **(D)** $2^{-1/2}\,T_0$

2. In a vertical upward field the electrostatic force on a pendulum bob is $mg/4$ upward. The period becomes:
   (A) T_0 **(B)** $\dfrac{\sqrt{3}}{2}T_0$ **(C)** $\dfrac{2}{\sqrt{3}}T_0 \approx 1.155\,T_0$ **(D)** $\dfrac{4}{5}T_0$

3. A particle $(q,\,m)$ enters a field region of length L with speed v_0 perpendicular to E. The angle of deflection on exit satisfies:
   (A) $\sin\phi = \dfrac{qEL}{mv_0^2}$ **(B)** $\tan\phi = \dfrac{qEL}{mv_0^2}$ **(C)** $\tan\phi = \dfrac{qE}{mv_0L}$
   (D) $\tan\phi = \dfrac{qEL}{2mv_0^2}$

4. Charges $+q, +q, -q$ are at the vertices of an equilateral triangle of side a. The field at the centroid has magnitude:
   (A) $\dfrac{3kq}{a^2}$ **(B)** zero **(C)** $\dfrac{9kq}{a^2}$ **(D)** $\dfrac{6kq}{a^2}$

4 Electric dipole

An **electric dipole** is a pair of equal and opposite charges $\pm q$ separated by a small distance $2a$. Its **dipole moment** is $\vec{p} = q\,(2\vec{a})$, a vector pointing from $-q$ to $+q$. Unit: C m. Total charge of a dipole is zero, but it still produces and responds to a field.

Field of a dipole (point dipole, $r \gg a$)

Position	Field	Direction
Axial (on the line through the dipole)	$E = \dfrac{2kp}{r^3}$	along $\vec{p}$
Equatorial (perpendicular bisector)	$E = \dfrac{kp}{r^3}$	opposite to $\vec{p}$

Both fall as $1/r^3$, faster than the $1/r^2$ of a single charge, because the two fields nearly cancel. Axial field is exactly twice the equatorial field at the same distance.

Dipole in a uniform field

- Net force $= 0$.
- Torque $\vec{\tau} = \vec{p}\times\vec{E}$, $\tau = pE\sin\theta$ (maximum at 90°, zero at 0° and 180°).
- Potential energy $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$. Stable equilibrium at $\theta = 0^\circ$ ($U = -pE$), unstable at 180° ($U = +pE$).
- Work to rotate from θ_1 to θ_2: $W = pE(\cos\theta_1 - \cos\theta_2)$.
- For small angular displacement it executes SHM: $T = 2\pi\sqrt{\dfrac{I}{pE}}$.

In a **non-uniform** field the dipole also feels a net force and tends to move towards the stronger-field region.